FIGURED ALGEBRA

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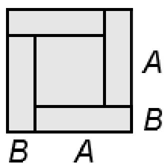
Extended Abstract
Ever since the work of Viète (1540-1603), algebra appears to be like a set of symbols and signs which are combined in a strict, regularized way. The succesful idea of Viète was to develop an efficient language to solve a great variety of mathematical problems. In former times, to begin with the ancient Babylonians, 3000 years before Viète, the language for solving a variety of problems had a geometrical background. Unknowns or variables were called length or width, products were called rectangles. Some historians call this 'geometric algebra'. For ages, this was the style of algebra. The Pythagoreans and their followers used dot patterns (figured numbers) to represent special numbers (for example triangular numbers). Euclid used line segments to illustrate his famous proof about the infinity of the sequence of primes. Al Khwarizmi and Cardano used squares and cubes to treat equations of the second and third degree.

So the algebra that we teach in secondary education has a long 'figured' history and it is my conviction that we can learn a lot of this history and do not forget the geometrical background and related visualizations in teaching algebra at school. It will deepen the insight in the algebraic rules and algorithms and will give the students a base of orientation and a possibility to reinvent procedures. In my talk, I will present different visual representations for algebraic situations, problems and procedures, meant for, say, 10- to 18-year-old students. Thereby I will borrow some examples from the history of mathematics and show some (unexpected) connections between different mathematical subjects.

As a foretaste I am offering this algebraic identity: $(A+B)^2 = (A-B)^2 + 4AB$,
which can be illustrated and understood by this picture:



Euclid treated this formula in the second book of the "Elements" by using a different illustration and formulated this in a more complicated way (for students in the 21th century), as follows: if a straigt line be divided into any two parts, the square of the sum of the whole line and any one of its parts is equal to four times the rectangle contained by the whole line and that part, together with the square of the other part. A possible exercise could be to explain the equivalency of this proposition with the algebraic formula above. This formula is powerful, because it can be used for instance:

- to find a general formula for solving quadratic equations,
- to find a rule to describe all possible Pythagorean triples,
- to prove that the geometric mean is at most equal to the arithmetic mean.

In the way that I have in mind, algebra education will be much more than learning and training a set of procedures in an automatical way. On the contrary, the techniques of algebra can be learned in an active and insightful way and can be experienced as a powerful tool to describe number patterns, to prove interesting numerical properties and to solve practical problems.

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