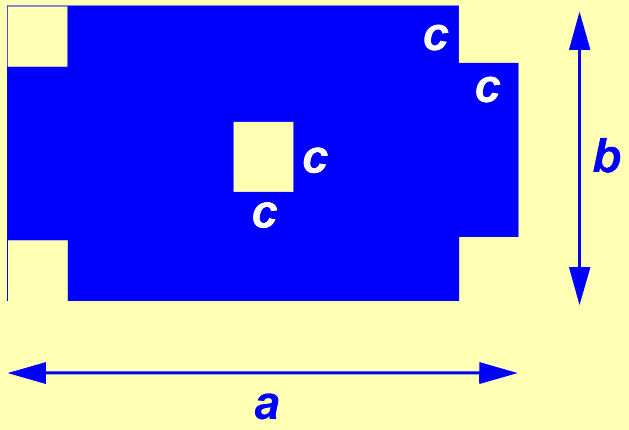
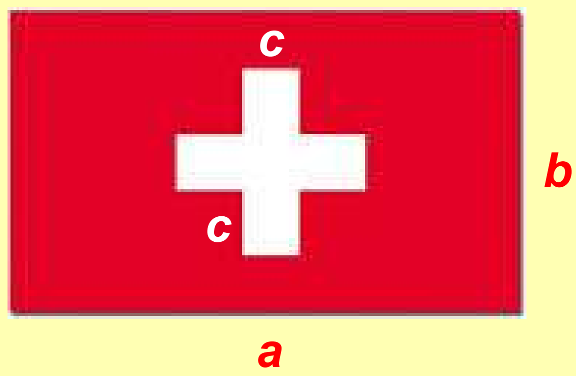
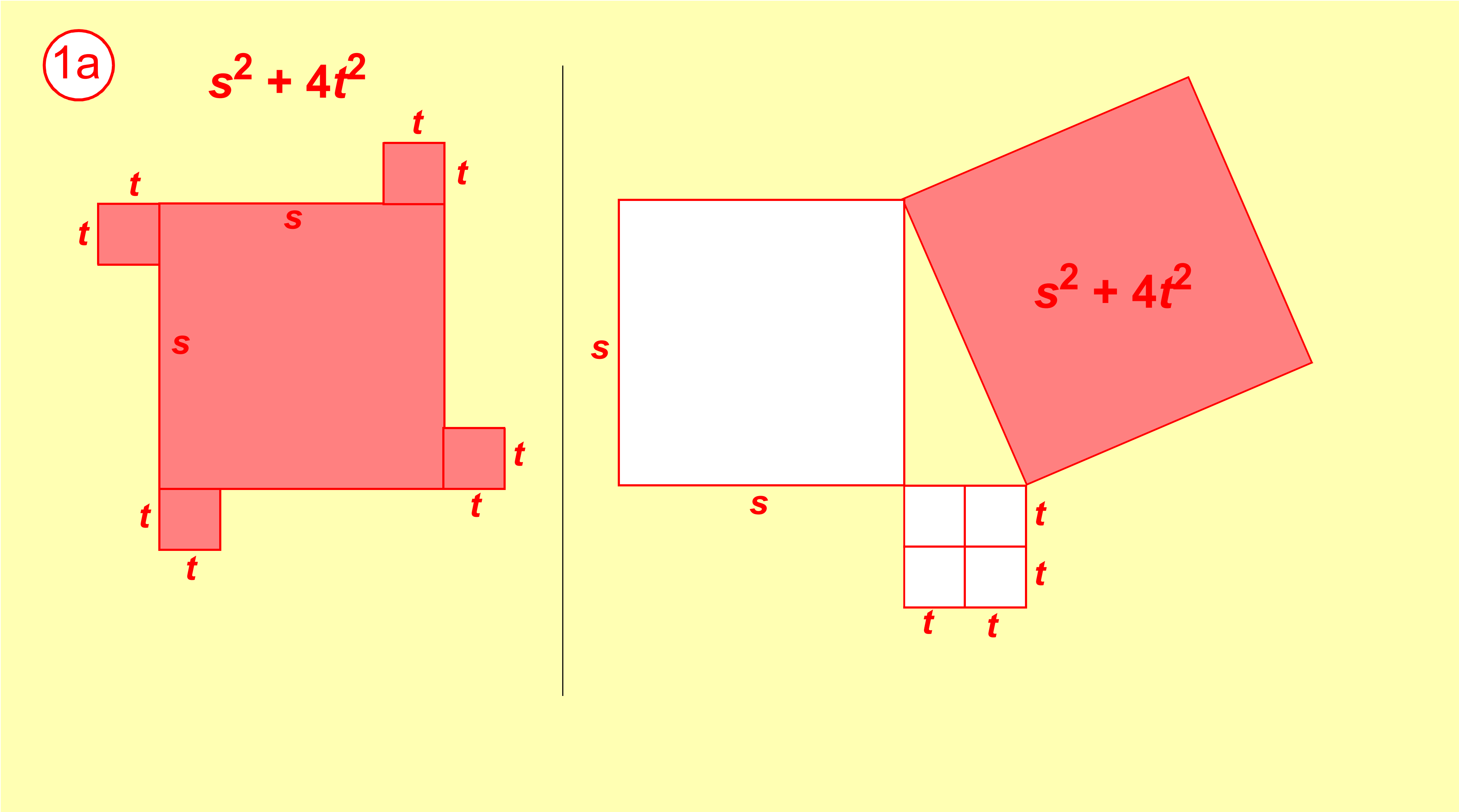
1a

$$ab - 5c^2$$

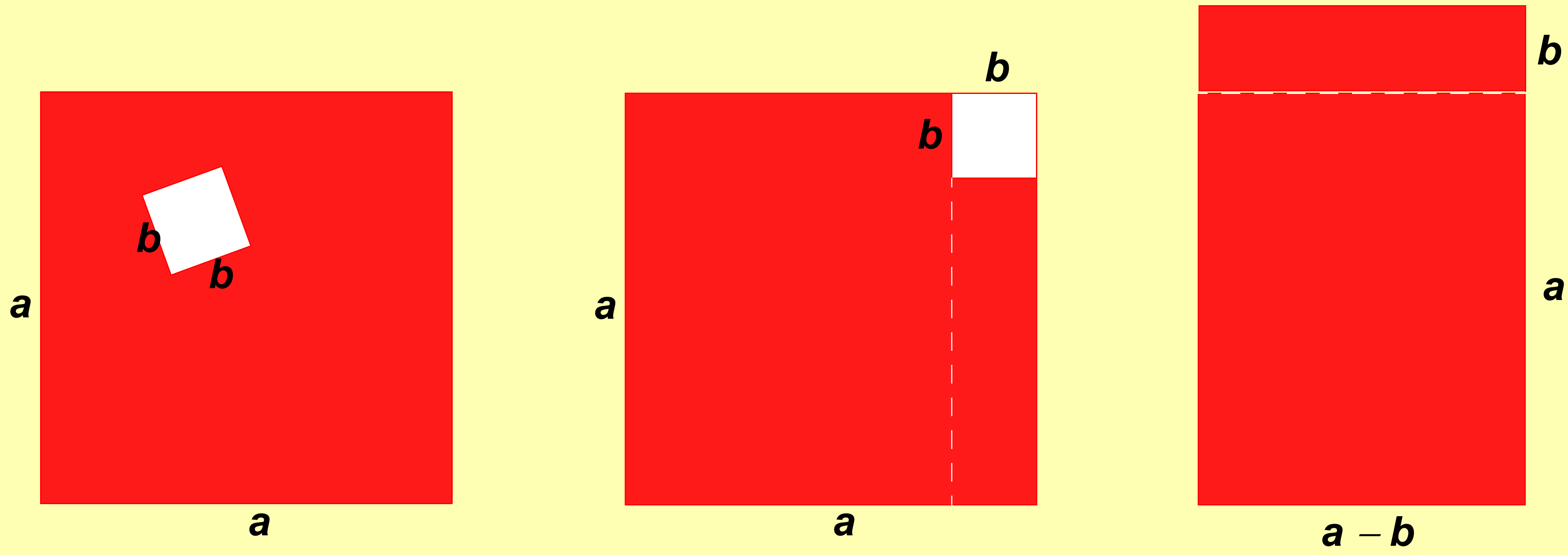


$$ab - 5c^2$$

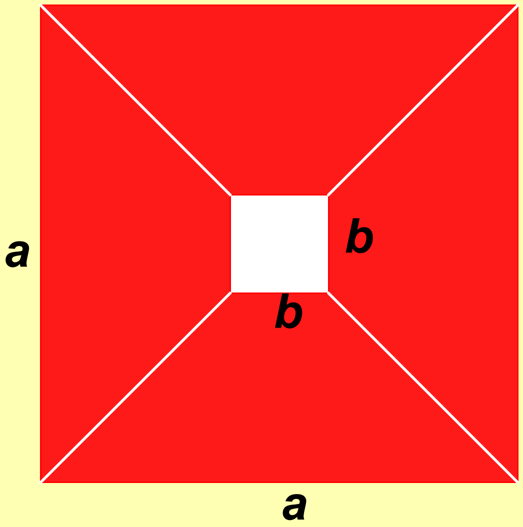


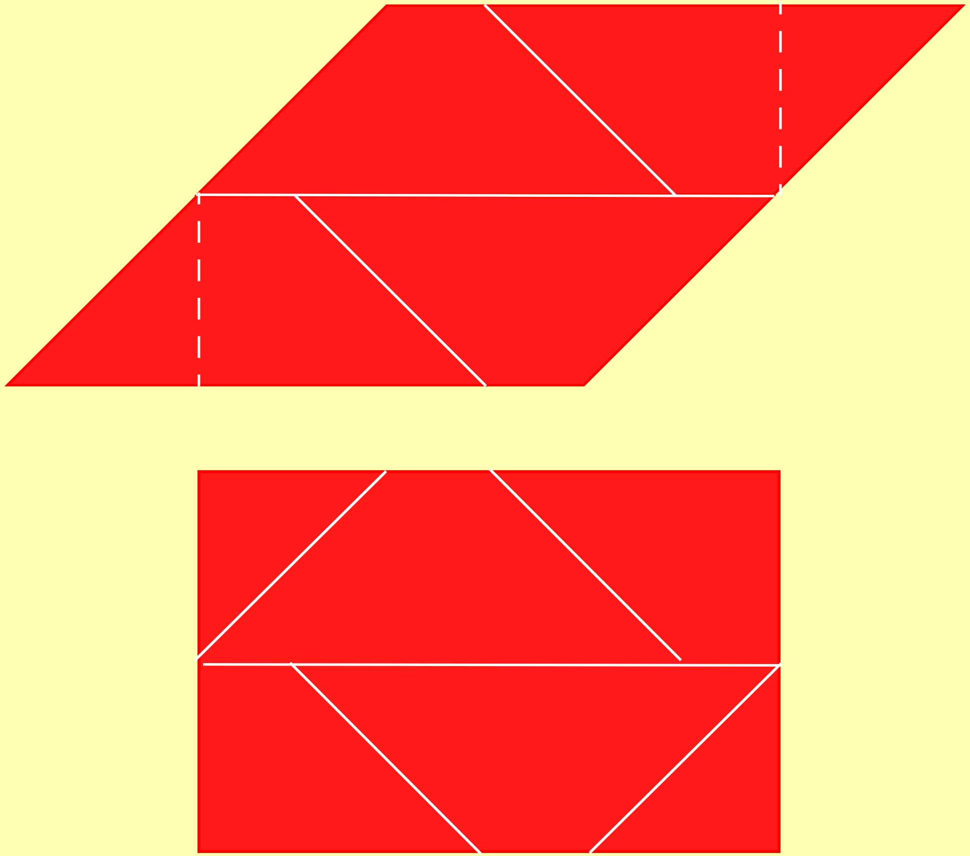

1a
s2 + 4t2
t
t
t
t
s
s
t
t
t
t
s
s2 + 4t2
s
t
t
t
t

1b

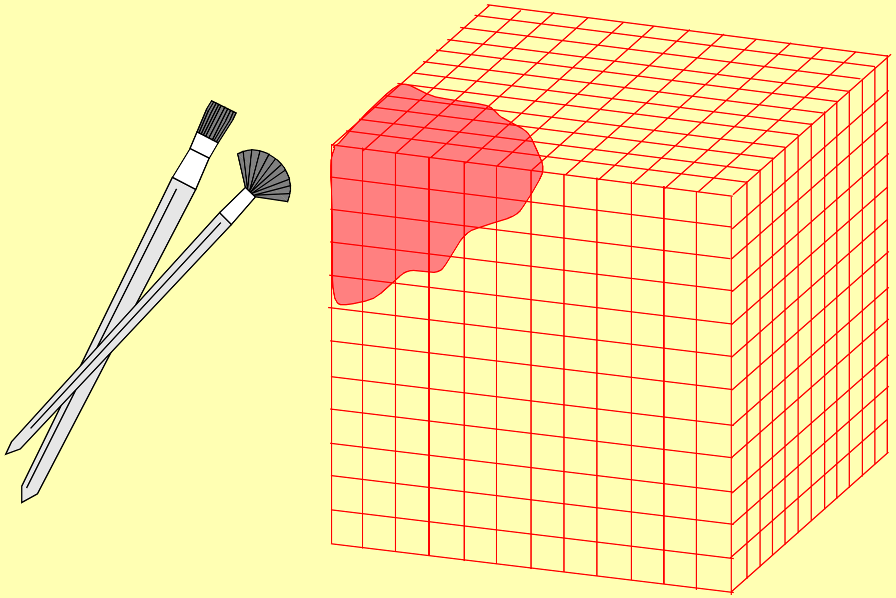


1b





2a



$12^3 = 1728$

# red faces	# bricks
3	8
2	120
1	600
0	1000
	+
	1728

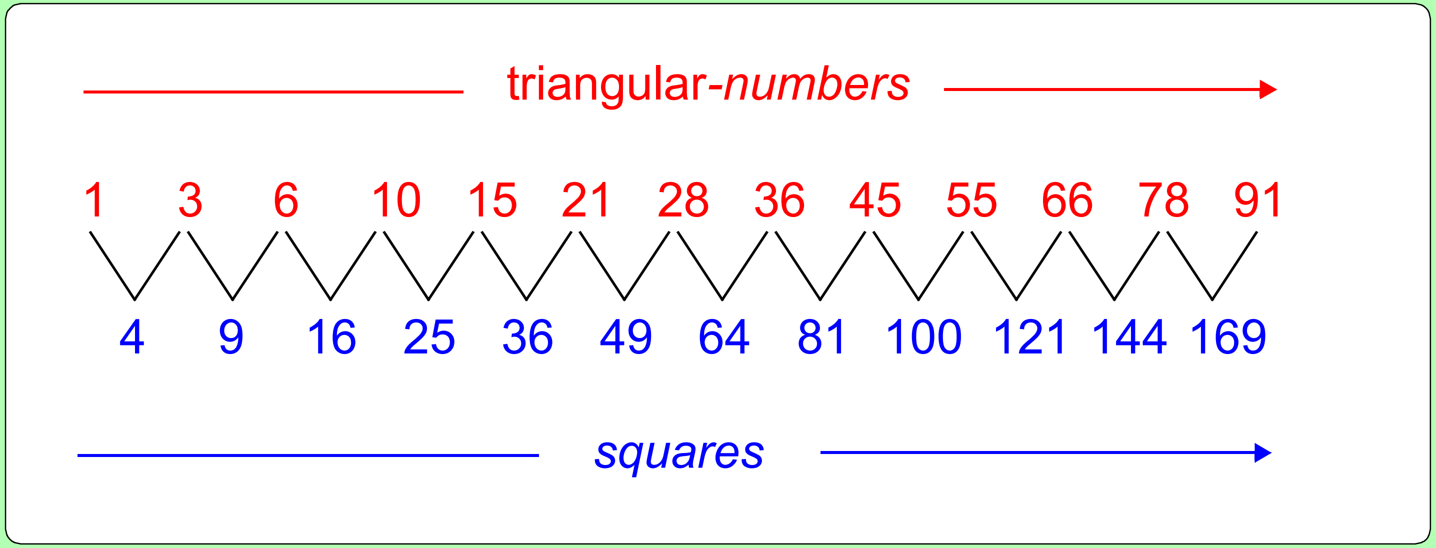
2b

cube of (2 + n) bricks

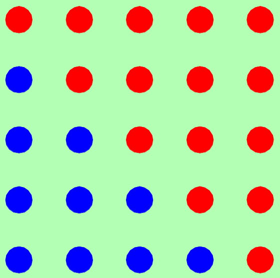
# red faces	# bricks
3	8
2	$12n$
1	$6n^2$
0	n^3

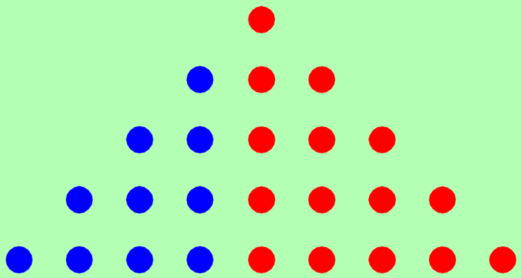
$$8 + 12n + 6n^2 + n^3 = (2 + n)^3$$

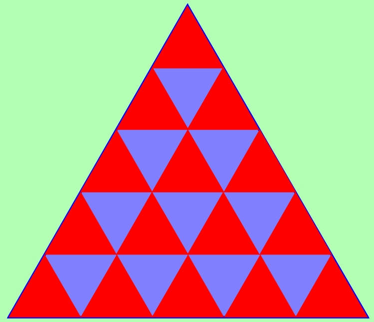
3


triangular-*numbers*
1 3 6 10 15 21 28 36 45 55 66 78 91
4 9 16 25 36 49 64 81 100 121 144 169
squares

③





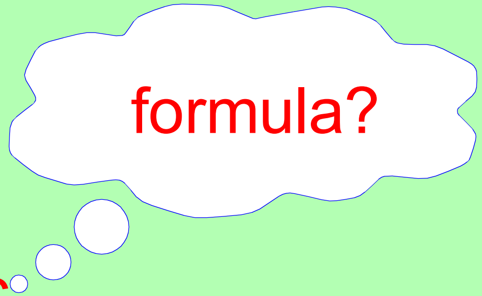


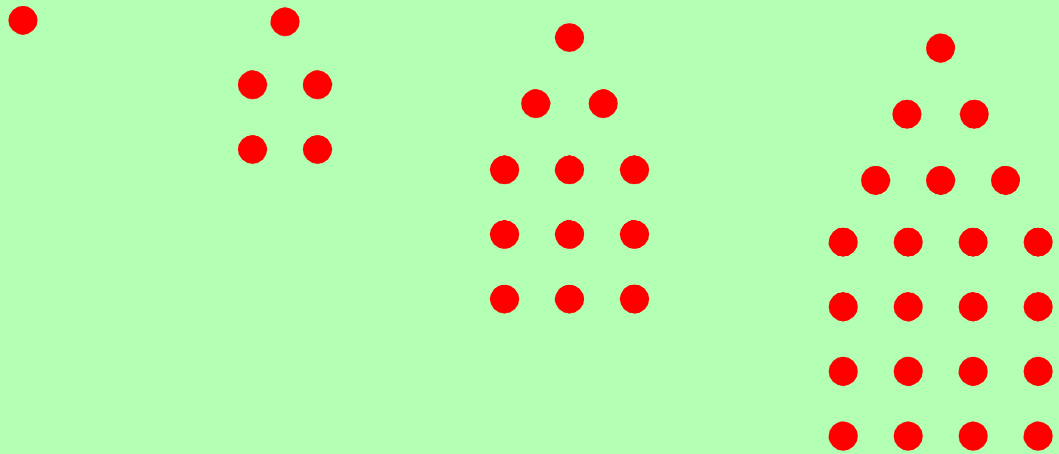
$$\Delta_{n-1} + \Delta_n = n^2$$

Algebraic proof $\frac{1}{2}n(n-1) + \frac{1}{2}n(n+1) = n^2$

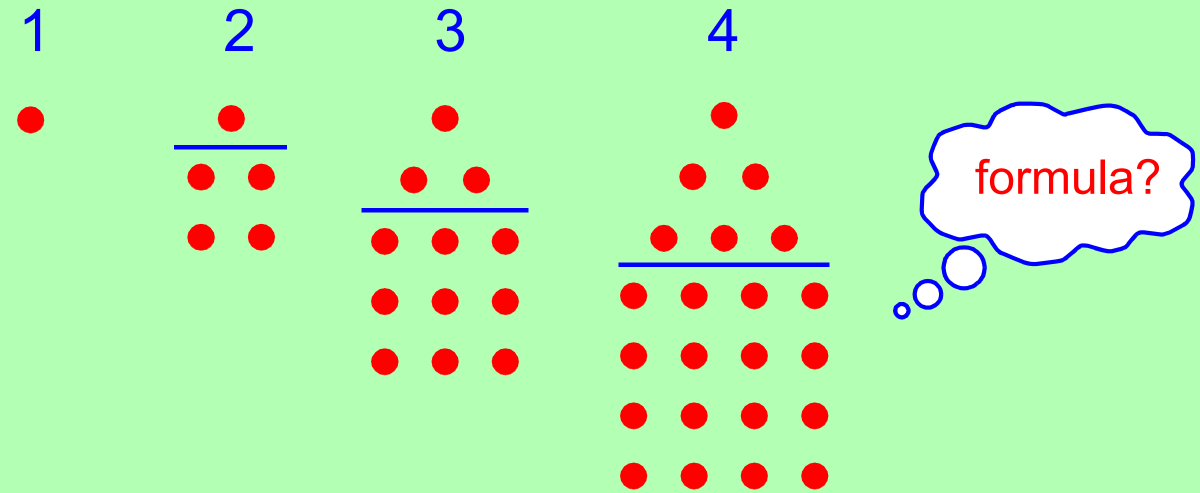
Pentagonal numbers

1, 5, 12, 22, 35, 51, etc.





4a



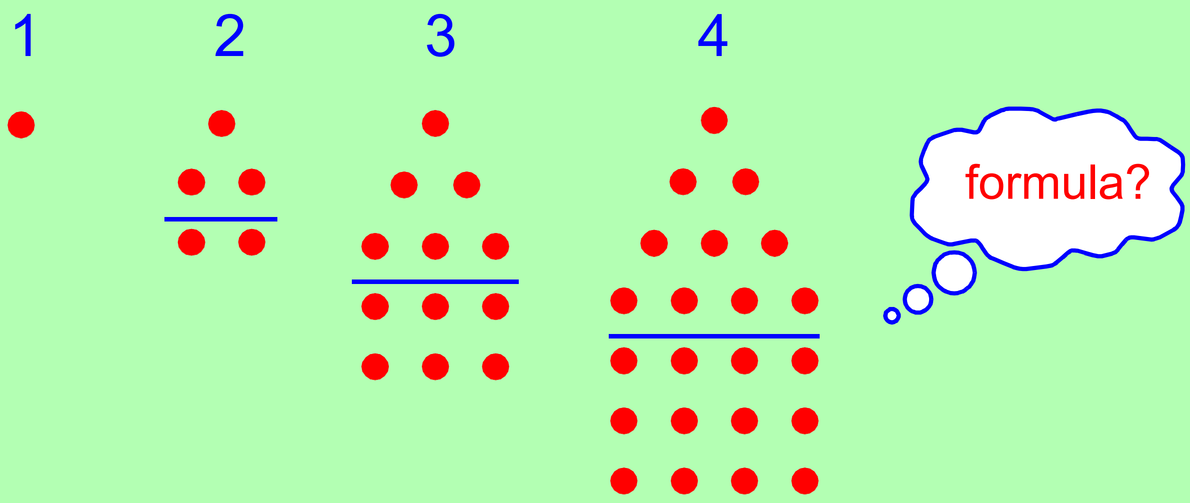
$$n^2 + \tfrac{1}{2}(n-1)n$$

↓

$$\tfrac{1}{2}n(3n-1)$$

or

4a



$$n^2 + \frac{1}{2}(n-1)n$$

$$\downarrow$$

$$\frac{1}{2}n(3n-1)$$

or

$$(n-1)n + \frac{1}{2}n(n+1)$$

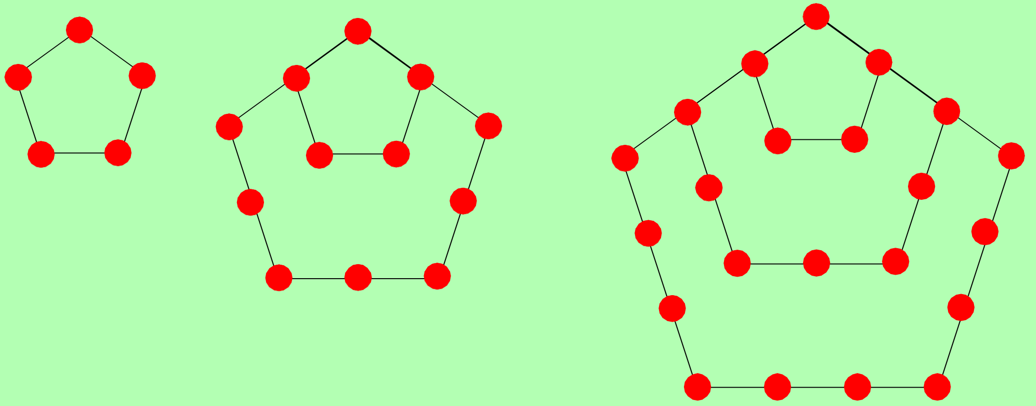
$$\downarrow$$

$$\frac{1}{2}n(3n-1)$$

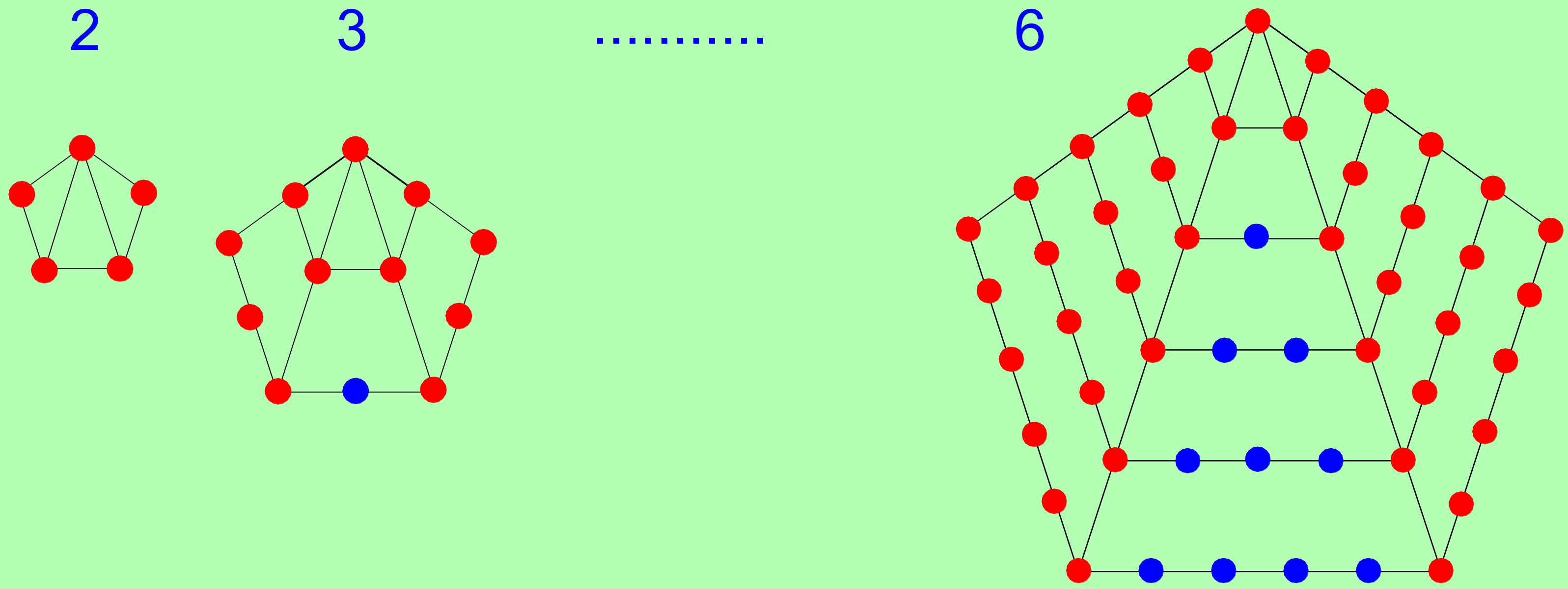
4b

Pentagonal numbers

1, 5, 12, 22, 35, 51, etc.



4b



$$2 \cdot \frac{1}{2}n(n+1) - 1 + \frac{1}{2}(n-2)(n-1)$$

$$\downarrow$$

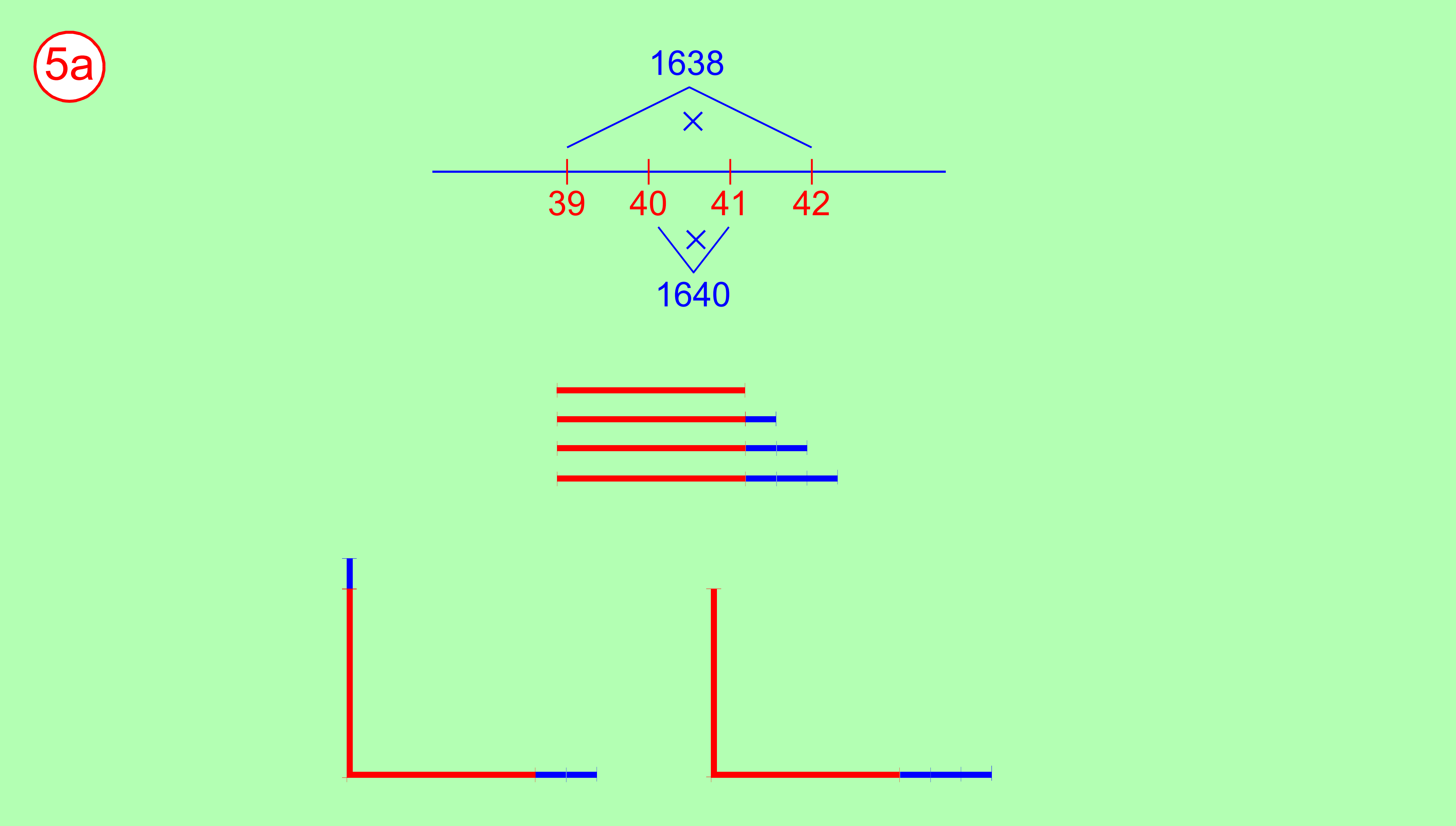
$$\frac{1}{2}n(3n-1)$$

3-Eck	$\frac{n(n+1)}{2}$	10-Eck	$n(4n-3)$
4-Eck	n^2	11-Eck	$\frac{n(9n-7)}{2}$
5-Eck	$\frac{n(3n-1)}{2}$	12-Eck	$n(5n-4)$
6-Eck	$n(2n-1)$	20-Eck	$n(9n-8)$
7-Eck	$\frac{n(5n-3)}{2}$	25-Eck	$\frac{n(23n-21)}{2}$
8-Eck	$n(3n-2)$		
9-Eck	$\frac{n(7n-5)}{2}$	m-Eck	?

Leonard Euler
from
Vollständige Anleitung zur Algebra

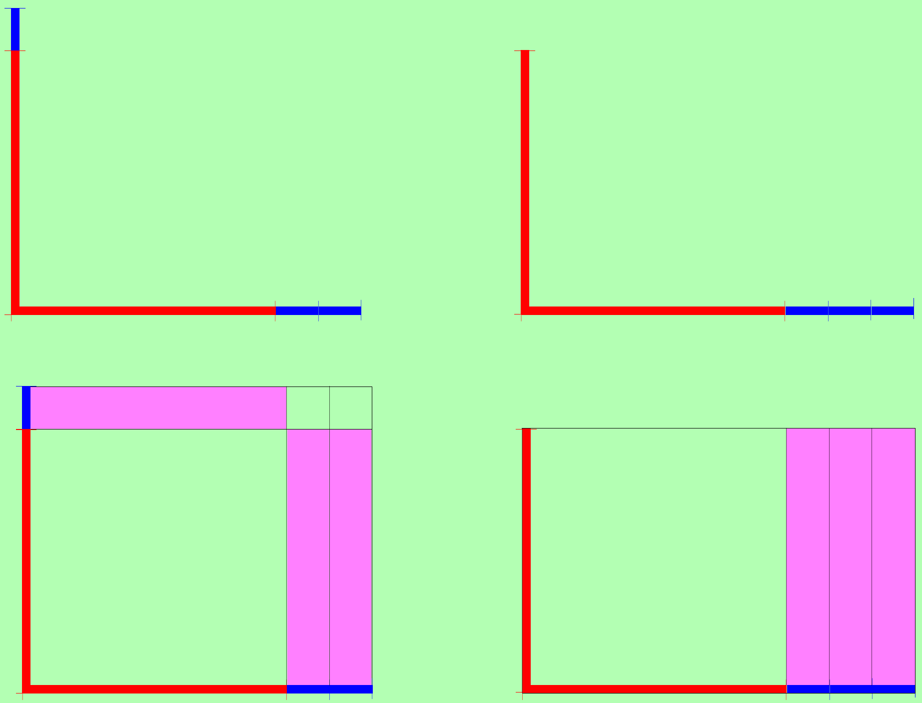
3-Eck	$\frac{n(n+1)}{2}$	10-Eck	$n(4n-3)$
4-Eck	n^2	11-Eck	$\frac{n(9n-7)}{2}$
5-Eck	$\frac{n(3n-1)}{2}$	12-Eck	$n(5n-4)$
6-Eck	$n(2n-1)$	20-Eck	$n(9n-8)$
7-Eck	$\frac{n(5n-3)}{2}$	25-Eck	$\frac{n(23n-21)}{2}$
8-Eck	$n(3n-2)$		
9-Eck	$\frac{n(7n-5)}{2}$	m-Eck	$\frac{(m-2)n^2-(m-4)n}{2}$

Leonard Euler
from
Vollständige Anleitung zur Algebra

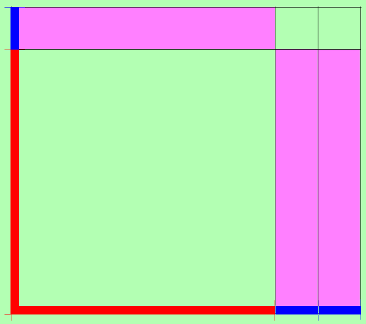


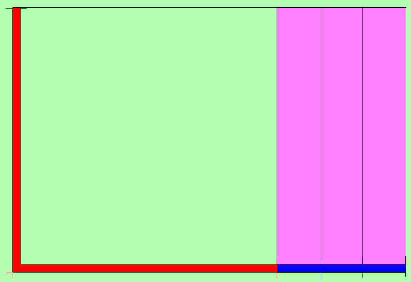
5a
1638
39
40
41
42
1640

5a



5a





$(n + 1)(n + 2)$

$=$

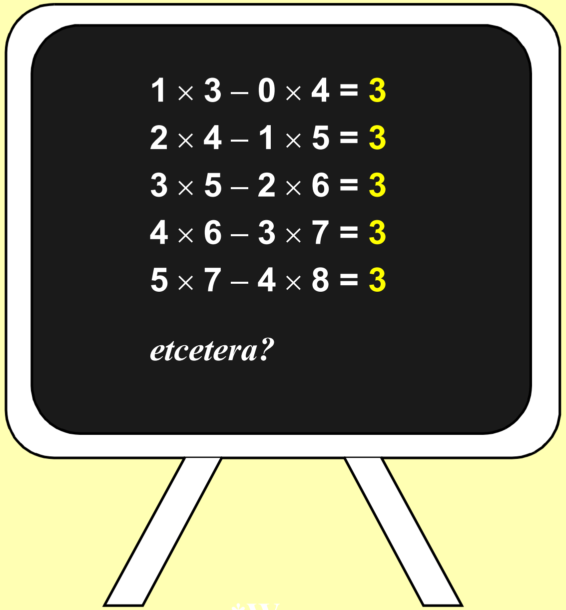
$n^2 + 3n + 2$

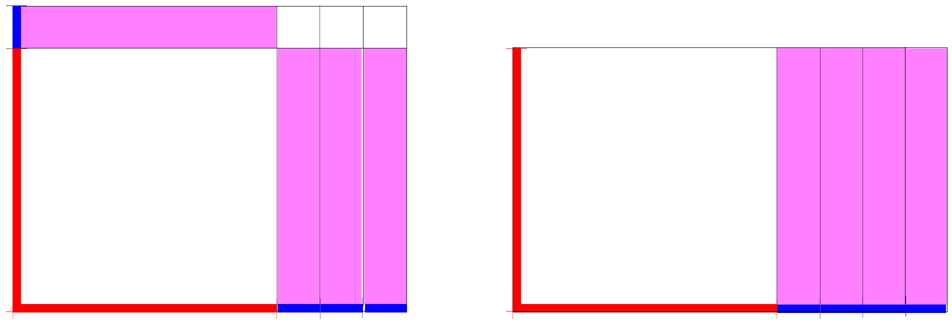
$n\,(n + 3)$

$=$

$n^2 + 3n$

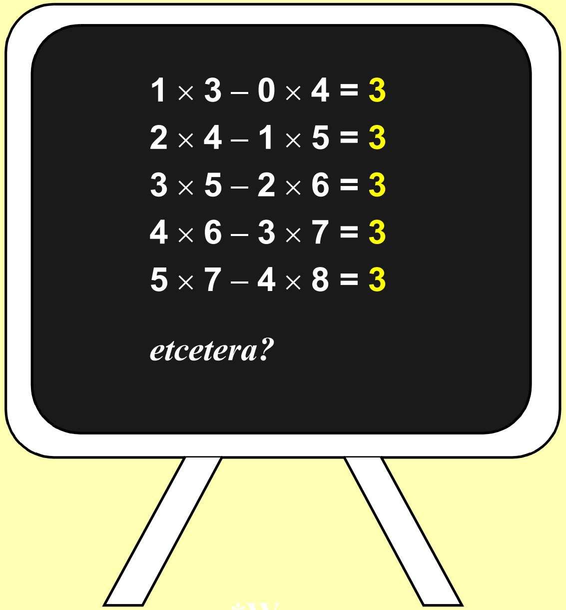
5b

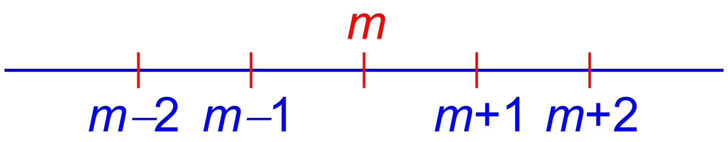




$$(n + 1)(n + 3) - n(n + 4) = 3$$

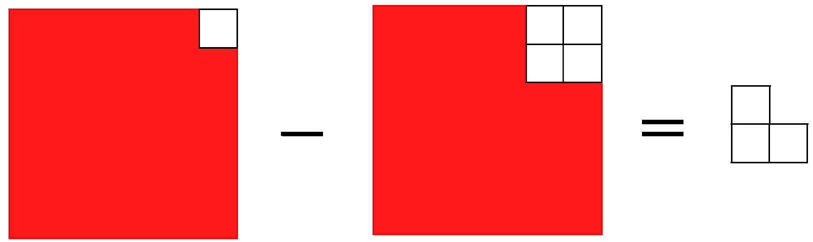
5b





$$(m-1)(m+1) - (m-2)(m+2) = 3$$

$$m^2 - 1 \qquad m^2 - 4$$



A teacher of mathematics has a great opportunity.
If he fills his allotted time with drilling his students in routine operations, he kills their interest, hampers their intellectual development, and misuses his opportunity.

But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.

George Polya
1887-1985