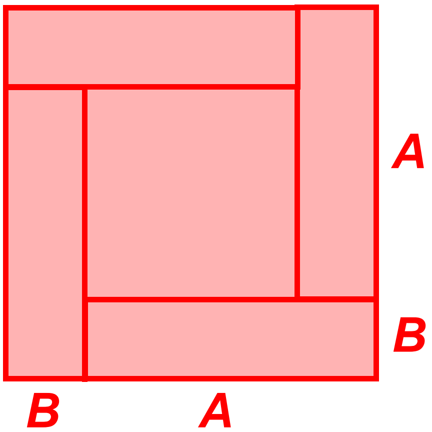
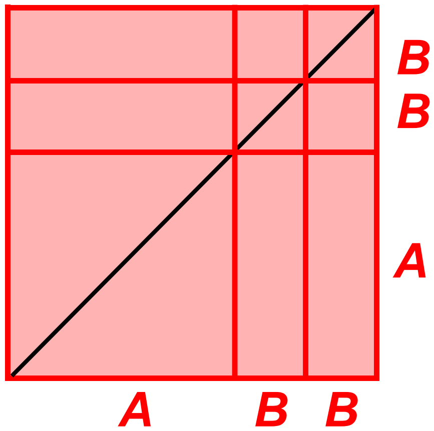
Workshop ‘Figured Algebra’ (KUPM 2018)

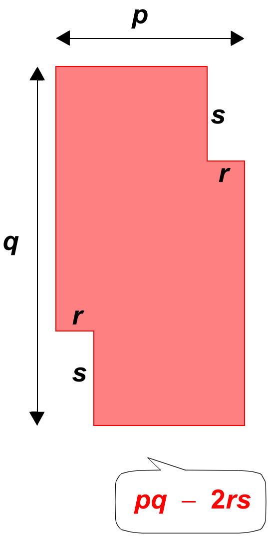
(Martin Kindt, *Freudenthal Institute, University of Utrecht*)

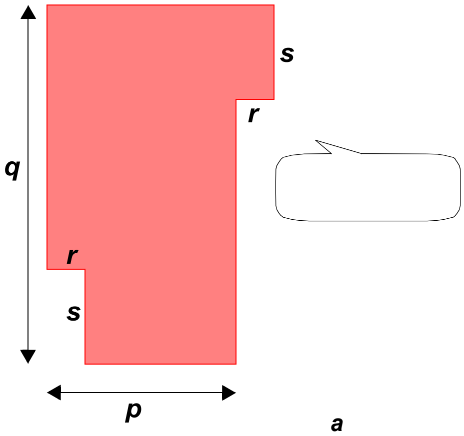


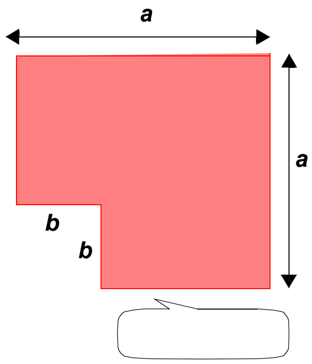


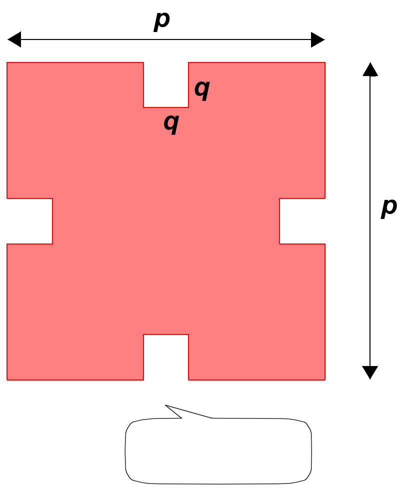
On the next pages you find some exercises in ‘Figured Algebra’.
It’s a good idea not only trying to solve the problems, but also discussing with each other about didactical issues and possibilities of these.
Together we will reflect on several solutions of these exercises.

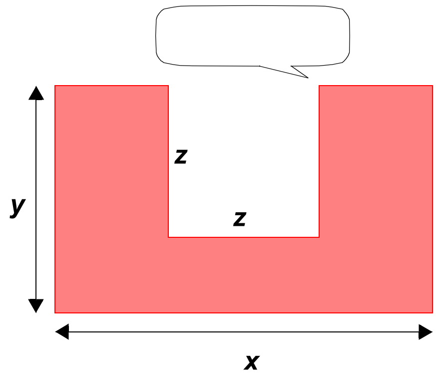
1a *Area-expressions*

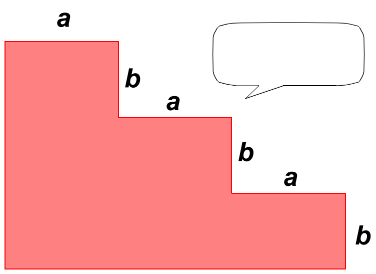






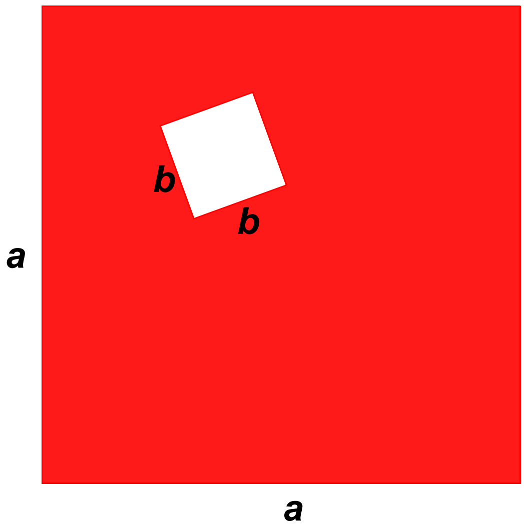






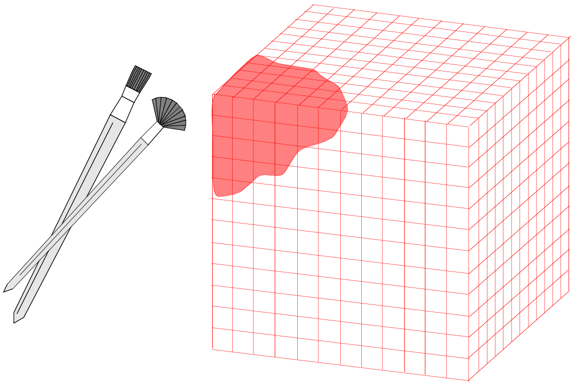
- Design a figure with area $ab - 5c^2$
- Also one with area $s^2 + 4t^2$

1b From a square with sides a is cut off a square with sides b.



- Draw a rectangle with the same area as the remaining red part.

2a Painting a cube



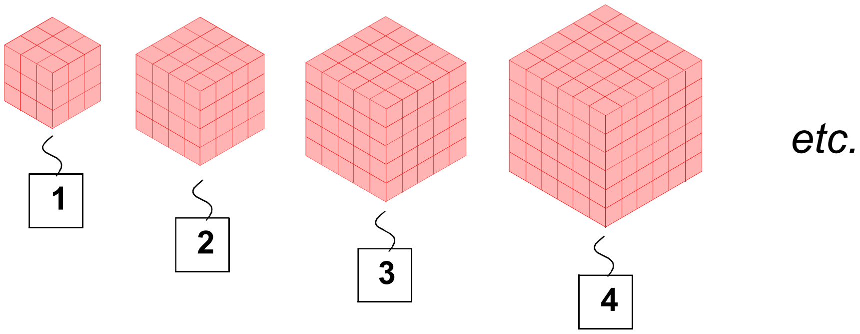
One cube ($12 \times 12 \times 12$ cm) is glued from little wooden cubes ('bricks') of 1 cm^3

- How many bricks are used?

The faces of the big cube are painted red.
There are bricks that get 3 red faces.

- How many?
- How many bricks get only 2 red faces?
- How many get only 1 red face?
- How many bricks are not painted at all?

2b A series of painted cubes



The first cube is 3 × 3 × 3 cm , the second one 4 × 4 × 4 cm, etc.
Look at the table below; n represents the number in the sequence.
You can read the value of n on the labels in the drawing.
The number of bricks with 3 painted faces, is called C_3
C_2 , C_1 and C_0 represent the number of bricks with 2, 1 and 0 painted faces. C_{tot} represents the total number of bricks.

n	C_3	C_2	C_1	C_o	C_{tot}
1	**8**	**12**	**6**	**1**	**27**
2					
3					
4					
5					

- Check the first row in the table and complete the table.
- Which regularities do you notice?
- Explain the formula: $C_2 = 12n$
- Explain the identity: $8 + 12n + 6n^2 + n^3 = (2 + n)^3$

3 Triangular numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...

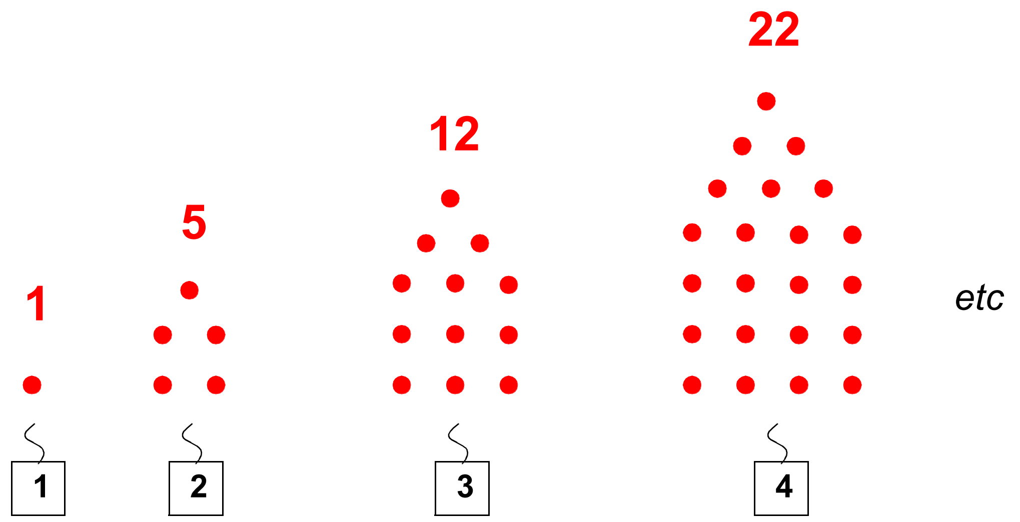
4 9

- Repeted add two successive triangular numbers.
  Which familiar numbers do you get?

- How to explain what you noticed by using pictures ?

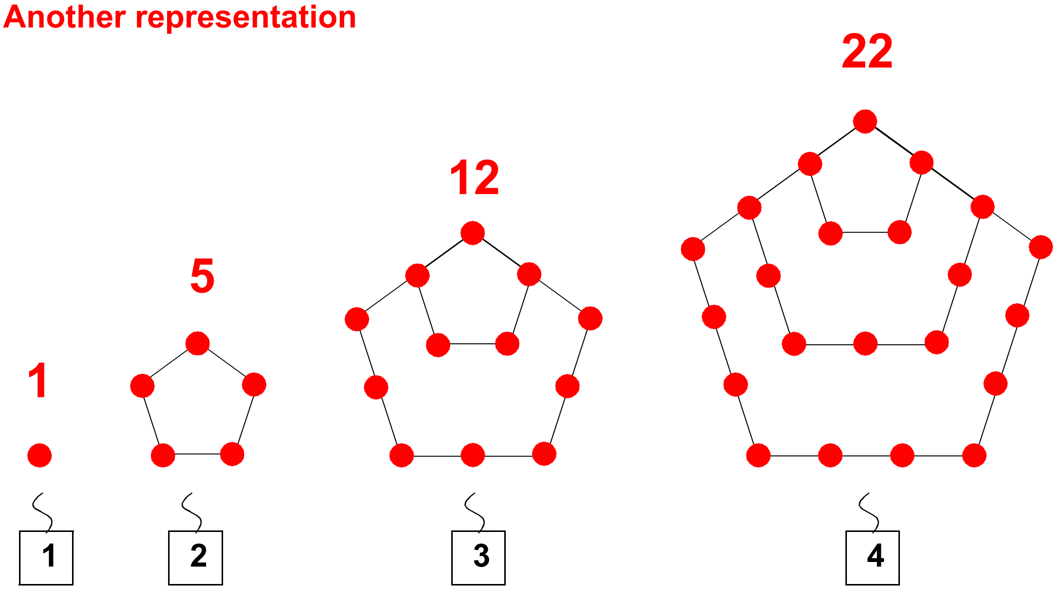
- How to explain this using formulas ?

4a Pentagonal numbers

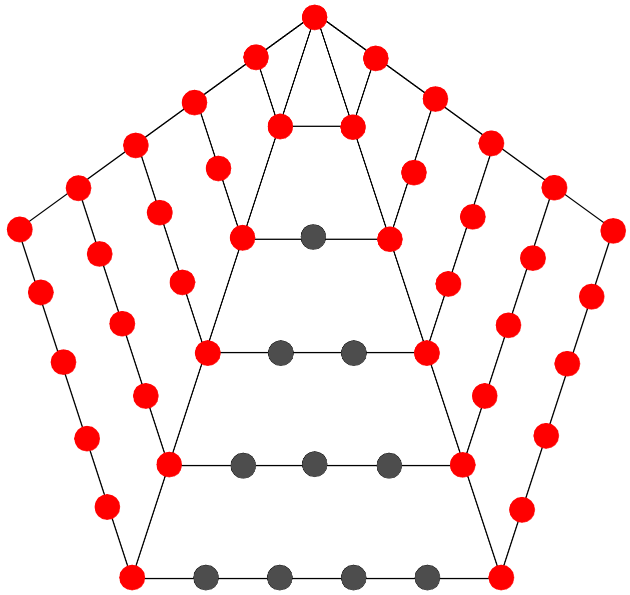


- Find a formula for the pentagonal number correponding with with $\boxed{n}$

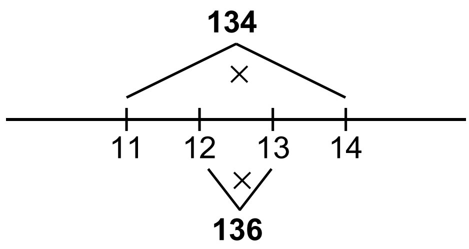
4b Another representation



- Find from this structure a formula for the pentagonal number corresponding with $\boxed{n}$



5a Four succesive numbers

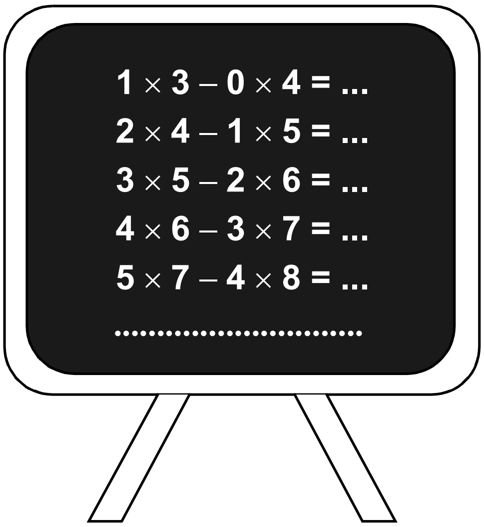


- Take any four successive natural numbers.
  Compare the product of the two inner numbers with the product of the outer ones.
  Repete this for other groups of four. Which rule do you expect?

- Prove this rule by using line segments to represent numbers.

- Prove this rule by using algebra.

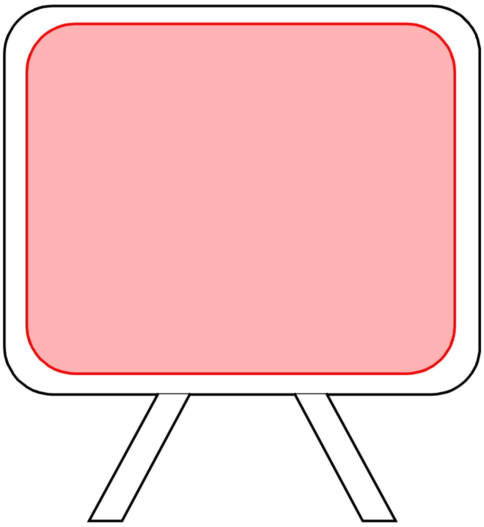
5b Pattern on the blackboard



- Which identity corresponds with this series?

- Draw a picture to explain the identity.

5c ‘Own production’



- Design your own series of calculations (with the same result on each line).

- Give an algebraic identity which corresponds to your sequence.